The Essence of Orientation - the Text

From my Travel Diary

...Finally, on the fourth day the Soviet helicopter took me to the oasis from which the caravan I was going to film would leave. The Master caravaneer was not very happy with the delay I had forced on him, but as it was the chief of his tribe who had authorized me, he expressed his discontent merely by hastening the departure. Barely out of the helicopter, still under the blades, I understood that I was going to miss a crucial phase in the process of navigation; as I could not film it, I will describe it to you:

There was a carriage with two horses and with two wheels, the wheels had a diameter of over two meters and drove a complex system of gears leading to a pointer in the guise of an effigy of Buddha or Confucius, but called Khan by everybody . Three men propped up under the chassis to raise one wheel, and a fourth man turned it to adjust the pointer to the direction indicated by wise man. I call him wise because of the length of his beard. When the direction of the khan was approved by the wise man, the other men lowered the wheel back to the ground and harnessed the horses. That done the caravan set out. After six days in a straight line through the Gobi desert - always following the pointer on the carriage - we arrived at the other side, and which is more, in time for the ritual of "fire and wind", a ritual where the participants must arrive guided by a pointer carriage. This was my first encounter with the ancestor of the compass.

From my Private Diary

When I was fourteen years old, still in Belgium, there was an exposition on the Far East, organized by Jesuits, in the top floor of a department store called the Big Bazaar. In this exposition, there were two objects which left me deeply impressed:

The first one was an ivory ball, perfect, with engravings all over its surface, regularly pierced by small holes, and through these holes one could see another ball inside, completely free to turn within the first one; this ball's surface again was completely engraved. So far the object already awed me, but really stupefying was the fact, that it actually contained seven spheres, each one completely free within the next enclosing one. I never found out for what the object was used, but that is not important, I never knew the meaning of this object, and that is much more serious.

The second object was a photograph, black and white of course (like sometimes wishes of poor monks remove the colour the reality, although), of a kind of carriage, half rotten and half buried in a archaeological excavation. The carriage had between its wheels a series of gears, or rather remains of gears. Adjacent to the picture there was a text explaining why it acted as a pointer carriage, a carriage provided with a mechanism made so that whatever trajectory one might chose to take with the carriage, the pointer always preserved the direction which one had selected. Being of an age where I started to care about girls but still played Meccano, I envisioned a mechanism with gears, based on a differential, which would meet the description. I promised to me to build one of them one day. Later, in Canada, I saw a very small article on this kind of carriage in the Quebec Sciences (November 71) and I repeated my vow: One day I would be done with one of them. This day now has arrived and passed; here is the carriage.

True Story

2634, yes! Two thousand six hundred and thirty four before Jesus Christ the emperor Huang Ti invented a carriage to allow his troops to advance in a straight line over unknown ground and in the fog. A certain Fang Bo is said to have built this "first" pointer carriage. Some see in Huang Ti "the father of China", who had "invented" a certain number of "mechanisms", systems of architectural constructions and of course military and political strategies.

Around 900 AC the Chinese discovered "the magnetic virtue of the floating needles", i.e. the compass, or the compass for navigation. Therefore, for approximately 3500 years, the pointer carriage could be used as instrument of navigation.

1588 AC Agostino Ramelli, an Italian engineer of the renaissance, comments on a pointer carriage used by the

Mongols in the Gobi desert.

Considering all, it is probably the first occurrence of the use of gears in the history of humanity, and a very cleverly made piece of mechanism, too.

Function

The principal idea is, quite simply, the following one: When a carriage with two wheels follows a curved trajectory, the wheel on the outside of the curve must traverse a larger distance than the wheel on the inner side. In extreme cases, one can imagine easily, the carriage can turn in place around one of its wheels, i.e. that one wheel remains fixed and the other one traces on the ground a circle which radius is the distance between the wheels (a geometric compass to some extent). In this case, one wheel turns on its axis and the other one does not turn at all. There is sort of a "measure" of the change of direction (of the carriage or its trajectory) in the difference of rotation of the two wheels. The idea is to take this difference of rotation and bring it back to a pointer which will "compensate" exactly for the change of direction of the carriage.

That is the idea, but how? There are several possible mechanisms, which has even been the subject of a contest. The mechanism that I chose, like that of the Chinese, uses a differential with four gears; but initially "the atom of the mechanism": Perpendicular gears.

Perpendicular gears are used because they are easier to build from wood and are less sensitive to alignment errors than parallel gears.

The differential is this mechanism which one finds in all cars, tractors, trucks, all the machines having two coaxial driving wheels (except the go-karts, but that is another story). It makes it possible for the two wheels to keep driving while turning at different speeds. In an automobile system, the engine (and transmission) turns the differential which, in turn, turns the wheels at (possibly) different speeds.

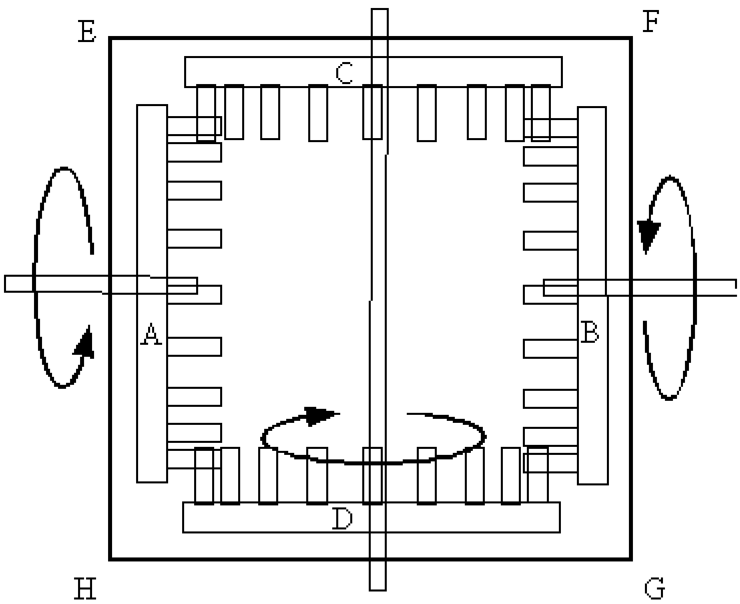
In a pointer carriage, the differential functions backward, i.e. the wheels possibly turn the differential which turns the pointer. The differential used here has the structure of a square cage, into which four gears are mounted on perpendicular axes. Two of the gears are fixed to their stub axles (they are called the planetary gears) and the other two gears turn freely on the same axle (they are called the satellites). Cage EFGH is supported by the axles of gears A and B, and it can also swivel around these axles.

EFGH: Differential cage

A, B: Planetary gears, locked to their axles, and their axles turn within the cage.
C, D: Satellites, free to turn around their axle, and their axis is fixed to the cage.

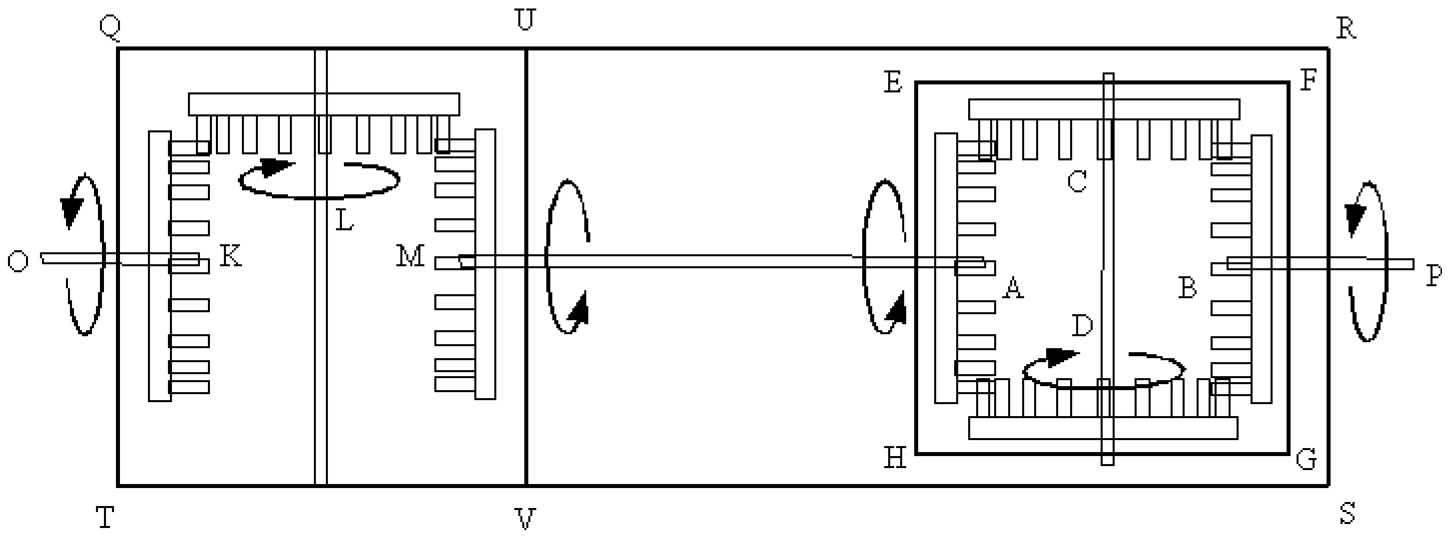
The four gears are all identical: Same diameter and same number of teeth.

If the cage is held stationary, and one planet gear is turned, let us say B, the other planet gear, A will turn in opposite direction. The two satellites, C and D, will also turn in opposite directions.



To visualize the movement, imagine to turn the planetary B as indicated by the arrow around its axle. The lower teeth, in the vicinity of the point G, will advance towards you while the upper teeth, in the vicinity of F, go away. This means that the teeth of right hand side of satellite D (region G) will advance towards you, driven by the teeth of B; as a result, the teeth on the left side of D, in the vicinity of H, go away from you; thus the lower teeth of A also go away from you, driven by the teeth of D. From this follows: "If the cage is fixed, both planetary gears turn in different directions".

As well can be said: If both planetary gears turn in different directions, the cage remains fixed and experiences no force for the same reasons. It is in this configuration that the differential is used in the pointer carriage. The differential is mounted so that when the carriage rolls in straight line, the planetary gears turn in different directions. Here's how the differential is installed in our carriage:



In a chassis QURSVT three gears K, L and M are placed, identical to those of the differential; gear K is fixed to axle OK, gear L turns freely on its axle (one could say that it is a kind of satellite) and gear M is fixed to axle MA. The axles OK, MA and LP are free to turn in the frame members, respectively QT, UV and RS of the chassis.

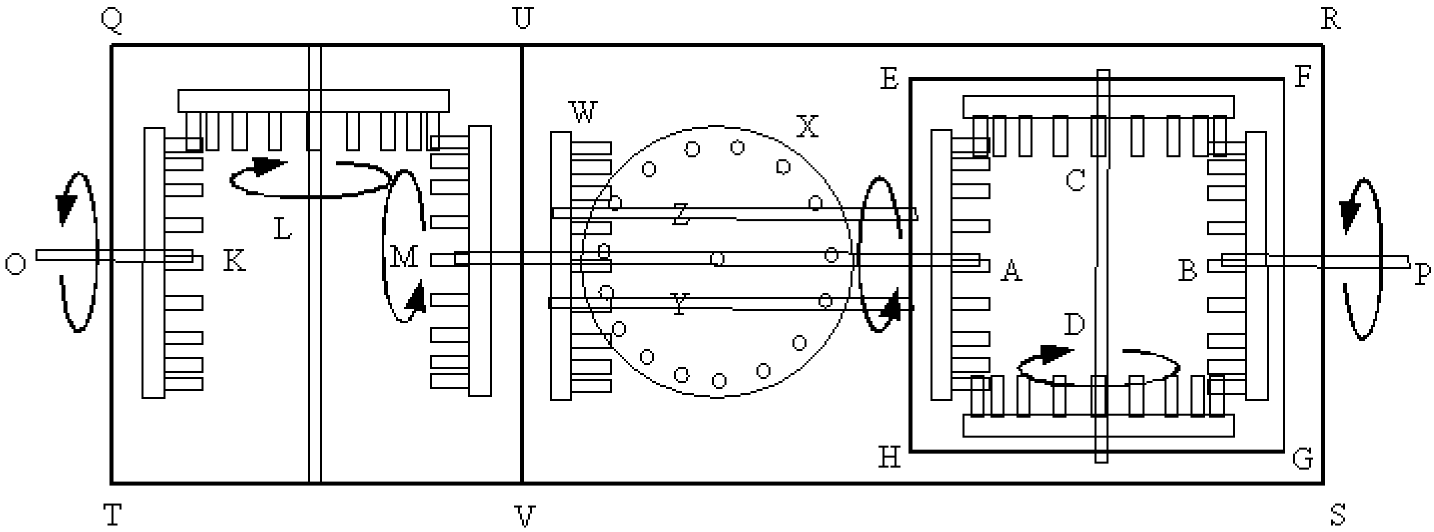
With the same principles that we encountered above, these three gears will achieve that if axle OK turns in one direction, axle MA will turn in the opposite direction. I call unit KLM the "reverser".

So that if the axles OK and PB turn in the same direction - for example because they are connected to the wheels of the carriage which advances in a straight line, as the arrows around O and P indicate (QUR would then be the front of the carriage and TVS the back) - the cage of differential EFGH will remain fixed, it will not swivel around the (imaginary) line AB.

So far for level one, level two shall be when the carriage does not roll in straight line. Let us imagine that there are wheels mounted to the axes OK and PB, one at O and one at P, each one locked to its axle. Let us also imagine that the carriage is tracing a curved trajectory, the two wheels still turning in the same direction but now at different speeds. Let us assume that the wheel at P turns more quickly (because it is on the outside of the curve) than the wheel at O; and to concretize the ideas let us assume that the gear K turns or "advances" four teeth while gear B turns six teeth. With reverser KLM, gear A will turn in the opposite direction of K but with the same speed. Thus we have the following situation: There is a differential ABCDEFGH which has its two planetary gears turning in opposite directions but at different speeds: While A in the vicinity of point H "moves backward" (away from you) for say four teeth, B in the vicinity of point G, "advances" (approaches you) for six teeth. The satellite D must "provide" six teeth at one side and four on the other; if it were four by four it would quite simply turn around its axle; but to achieve four by six it cannot turn only around its axle, it must additionally swivel around the teeth of H so that its centre advances towards you. One can make the following calculation: six minus four equals two. There are two additional teeth the satellite must "provide" at point G; if its axle (CD) advances towards you (with the end D) with a rotation which corresponds to one tooth, the side G of the satellite advances towards you by two teeth. The discussion is exactly the same for the satellite C but away from you for the same angle. This is exactly what the differential does: When both sides turn in opposite directions at different speeds, the axle (CD) of its satellites turns (D towards you) at half the difference of the speeds: The half of six minus four equals one! It is the angle corresponding to one tooth. Some quick remarks: If the two wheels turn at the same speed, the differential turns at half the difference of two speeds; as two speeds are equal, and than the half of zero is zero, the differential does not turn at all around the axles of A and B; we have level one again, which is only one particular case of level two.

Good, now to level three: The pointer.

Up to now there is a measure for the difference of rotation of the two wheels in the rotation of the differential. It is now necessary for us to find out and understand how the difference of rotation of the two wheels relates to the change of direction of the carriage.



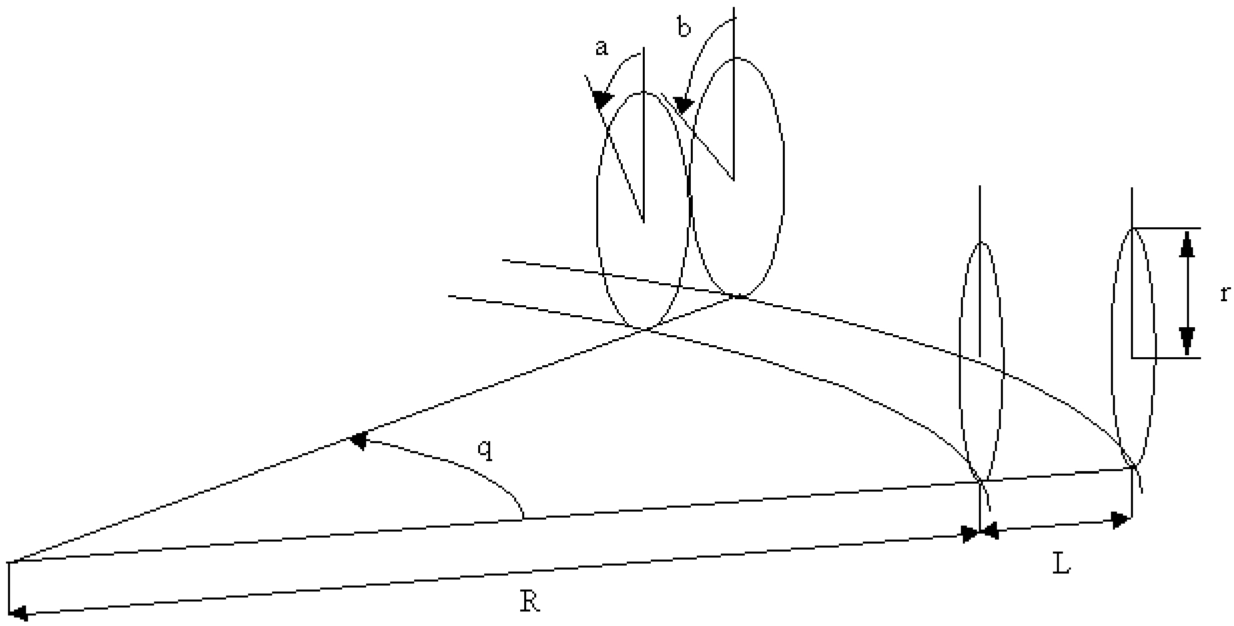
A Gear W is installed, which turns freely on the axle MA, and which is fixed by the "bars" (Y and Z) to the cage EFGH of the differential. Unit WYZ is used to "bring back" the rotation of the differential to the gear X, locked to the pointer. Gear X is mounted on a vertical axle, in the schemes the carriage is looked upon from the top downwards. The reason for this fix is as follows: Assume that the carriage advances (in the direction from V towards U) while turning towards the left, its wheels turn in the direction indicated by the arrows around O and P; the wheel at P turns more quickly than the wheel at O; side D of the differential advances towards you, which means that the teeth of side V of gear W advances towards you equally; and gear X turn "clockwise" (unfortunately an expression which undoubtedly will disappear because of the famous clocks known as "digital"). I.e. when the carriage turns left, the pointer will turn towards the centre line, so compensating for the change of direction of the carriage; it is still necessary for the "compensation" to be exact: The pointer (X) has to turn the right angle.

Let us imagine that a couple of chassis complete with reverser, differential and pointer as in this schema are manufactured. Pairs of wheels of all kinds of dimensions are installed and the movements of the pointer for a single trajectory of the carriages (say of a quarter of a circle) are observed. The direction of the carriage having changed by 90 degrees, the pointer must turn 90 degrees too. The correct direction of rotation of the pointer is guaranteed by design.

From our experience we know, when the carriages all traverse the same distance, the "smaller" wheels must make more turns; when they are "larger", they must make less turns. However our differential (and our pointer) are sensitive to the difference of rotation of the wheels; if the wheels are too small the pointer will swivel too much and if the wheels are too large it will not swivel enough. For a chassis of a given dimension there is thus an appropriate diameter for the wheels.

This not all. Let us now imagine that a series of chassis of different widths (distance between the wheels O and P) are made, and that the behaviour of these carriages with wheels of constant diameter is observed. For a narrow chassis, the difference between the distance covered by the inside and outside wheels will be small, therefore the difference of rotation of the wheels will be small and the pointer will turn only "a little"; for a wide carriage, the difference of covered distance for the two wheels is "large", the difference of rotation is "large" and the pointer compensates "too much". For a given wheel diameter there is thus an ideal distance between the centres of the wheels. Altogether what counts in reality is the relationship between the diameter of the wheels and the distance between their centres. It can be found by experiment but also by calculation, and the result is one! Quite simply. It is necessary that the distance between the wheels is equal to the diameter of the wheels so that the rotation of the pointer exactly compensates the change of direction of the carriage.

A Little Calculation



Let here be: L the distance between the wheels; r the radius of the wheels and R the radius of curvature of the trajectory (in a plane). Let us assume that the carriage traverses an angle q, in radians, on its trajectory. This angle q at the centre is also the angle of change of direction of the carriage, the angle between the two tangents to the trajectory at start and end of the angle q. The wheels then turn by an angle of a for the inside wheel and of an angle b for outside one. As the wheels roll without slipping, the travelled distances can be written as:

$$R\,q = r\,a \quad \text{and} \quad (R + L)\,q = r\,b$$

which gives:

$$q = r\,(b - a) / L$$

In addition, as the differential (and the pointer) turns half of the difference of the rotational angles of the wheels, if q is the swing angle of the differential (and pointer):

$$Q = (b - a) / 2$$

and we want Q = q, we get:

$$2\,r = L$$

The distance between the centres of the wheels needs to be equal to the diameter of the wheels, which is a configuration we find in the majority of the Chinese and Mongolian carriages.

If the carriage does not fulfil the last equation it is necessary to introduce a correction factor which adjusts the compensation of the pointer corresponds exactly to the change of direction of the carriage; this the role of the transmission made by the chains on our machine.

Let us assume that

$$2r / L = F$$

instead of

$$2r / L = 1$$

Reworking the equations produces:

$$r = f L / 2 \quad \text{and} \quad q = f (b - a) / 2$$

but the expression for Q (it is a property of the differential to be independent of the dimensions of the wheels or the chassis) needs not to be changed, so that to have Q = q it is necessary to cleverly multiply Q by f. This is what we chose to do by multiplying the angles a and b by f through the chain transmissions between the axles of the wheels and the mechanism.

There is another reason which pushed us to introduce transmission chains into our carriage: The axes of the mechanism could - for purely geometrical reasons - not be made from ¾ inch dowel, and as ¾ inch axles were necessary to securely carry wheels of around fifty centimetres in diameter, it was necessary to construct wheel and gear axles differently.

Technical Details

Wood

Chassis and mechanism: Beech

For the gears: "Pau Amarillo", which is a very heavy and very hard African wood (sounds like the name of somebody, which is probably the case, but that's what it is called); the teeth and the axes are in "goujons de bois franc du Quebec" (trade name) of 5/16 inch in diameter; there are 2,6 meters of this stuff used for the teeth of the gears and the axes in the mechanism.

Chassis bearing (bottom): Wild cherry tree; the axle of the wheels is in "goujons de bois franc du Quebec" of ¾ inches in diameter.

Wheels: Pine

Pulleys of the chains: The centre is in plywood (½ inch), the sides are red oak.

The pointer: built from the remainders of various drinks.

Assembly pins, either ¼ inch pins or bamboo, depending on diameter.

Critical Dimensions

Track (distance between the centres of the wheels): 57 cm

Diameter of the wheels: 44 cm

Gears: All in all 9 identical gears of 12 teeth. The plates of the gears measure 84 mm in diameter, the radius of the circle of the centres of the teeth is 34 mm.

The pulleys of the chains have 44 teeth on the axis of the wheels (bottom) and 57 teeth on the axis of mechanism, and that's not by chance.

Useless Details

There are 12 ball bearings (5/16" bore, 13/16" diameter) in the mechanism and 4 ball bearings (¾" bore, 1,5" diameter) for the axles of the wheels.

The "tires" are made from 14" bicycle tire tube, cut open and glued to the wheels.

The chains are usually used not to lose the plugs of bath-tubs or basins; they can also be found at window blinds sometimes. *This new use was invented by us.*

Weight: Approximately 6,9 kg, so it's like "Science on Tour".

The number of teeth for each wheel of the chain transmission: Valmont and me improvised here, they were not calculated, but fabricated to fit.

Geometric Properties of the Pointer Carriage

We can still go further with our chariot and roll it on something different than a plane surface; for until now we assumed (implicitly) that the carriage rolled on a plane.

Let us imagine that the carriage rolls on a sphere, a sphere whose radius is not too large nor too small relative to the radius of the wheels and the distance between the wheels of the carriage. Note that the constraints given here do not apply to the Mongolian carriage in the Gobi desert: It is true that the Gobi desert is spherical, but the radius of the earth is about 6000 km, i.e. 6000000 m and the diameter of the wheels is about 2 m - much too small to make the discussed effects observable.

Then, let us imagine that our carriage rolls on a sphere with a diameter of - say - ten meter. If the two wheels do not turn with the same speed, the pointer changes direction, so that if the direction of the pointer is followed all the time, the two wheels will necessarily traverse the same distance. That means that on the sphere the two wheels will trace arcs of circles of identical length. Just to give an idea: Let one wheel traverse on each side of the equator. The equator is only a particular case of what we call a "great circle". Actually all meridians are great circles. Through every point on a sphere we can have an infinite number of great circles. They are the biggest circles we can draw on the surface of a sphere. None of the parallels (except the equator) is a great circle.

The great circles have the following interesting property: If you take two arbitrary points on a sphere, the shortest distance on the sphere is an arc of the (one and only) great circle which passes these two points. All other lines on the sphere which pass these two points are longer. As an exercise in visualization, you need a globe and a rubber band; if you stretch the rubber band over the sphere, the band tends to follow an arc of a great circle and this will always be the shortest distance between its end points. These great circles are of course fundamental for navigation (terrestrial or maritime). The Arabs knew that around the year 700 AC. Because they had to turn to Mecca for prayer, their astronomers knew how to calculate the direction of the great circle through any given place and Mecca. *It is not true that the earth is flat* since Erathostenes, who had already in 250 BC measured the radius of the earth. The uneducated were unaware of it, but the scientists knew it.

Let us return to our rubber band and our globe. Tighten your band between Montreal and Paris and look where it passes: It marks the shortest distance between the two cities, it passes somewhere south of Greenland and Iceland, not further north that Montreal and Paris; that's why the planes follow this kind of trajectory - it is shorter.

Continue to hold your rubber band between Montreal and Paris and and look at the intersection of your band and any meridian. If you observe the angle attentively and compare it to the angles at the other meridians, you will find them all different.

If you stretch your rubber band between Montreal and Paris on a flat map of the world (a map where the meridians are parallel straight lines), and if you examine the angles between the meridians and the band, you will find them all identical. As Korzibsky said: "The map is not the territory".

And as the magnetic compass (correctly compensated for the local deviation) gives the direction of the meridian in any given place, if you want to follow a large circle (other that the equator) on the ground it is necessary to sail with a variable course. Our machine however correctly regulates the pointer to the direction at departure, and by following it will trace only the large circle; it is much better than a compass!

This Mongolian carriage is really an extraordinary navigation instrument, superior to the magnetic compass. Unfortunately you cannot make much use of it, for the dimensions of our planet would require wheels of at least fifty kilometre diameter to be sensitive to the roundness of the earth. Our planet is too large, but on an asteroid or a small planet, which knows?

You can roll the carriage on another things than a sphere. Even on non-spherical surface it maintains its property to point the shortest path between two arbitrary points of the surface. It is a geometric tool. These paths are called "geodesic". For example, the geodesics of a plane are straight lines, the geodesics of the sphere are the great circles, the geodesics of an ellipsoid are ellipses or circles respectively... The pointer carriage follows or traces the geodesics on arbitrary surfaces (as long as the dimensions are compatible of course).

You can still fantasize on a little. Einstein's gravitation works as follows. A 3-dimensional space is taken (instead of a surface, with two dimensions); this space has geodesic lines (it is called Euclidean). When one places a mass in this Euclidean space, the mass deforms the space so that the geodesics intersect no longer at right angles, the geodesics become curves (the space is called warped or non-Euclidean); the general relativity calculates these curves and the physical dead matter follows these geodesics spontaneously. In particular, the light follows the geodesics: Light travels in a straight line if and only if space is Euclidean. This is no theory, it was verified experimentally by Eddington in 1919. In fact, I lied to you a little bit: Einstein's gravitation works in a 4-dimensional universe, three of space and one of time, but as it is too difficult to imagine, I told you the story in three dimensions. In four it is almost similar, just a little more complicated. On the principle level that does not change anything.

The pointer carriage has one more spectacular geometric property. Let the carriage roll on a plane surface and force it to make a trajectory which is a closed curve, i.e. it starts from a point, makes a loop (anyhow) and returns to the start point. As the pointer maintains its direction all along the way, the direction of the pointer on arrival is exactly (with the precision of the instrument) the same as the direction of the pointer on departure. The difference of the covered paths of the two wheels will have made the pointer take one (or more) full turns. Good.

Now, if you drive a loop with the carriage on a non-planar surface (but "continuous" and in conformity with the dimensions of the carriage), the difference of the covered paths of the two wheels will be different from that for a plane surface, so that the pointer on arrival will be in a different direction than on departure. The difference between these two directions depends on the "non-planarity" of the surface, the difference (between the two directions) corresponds directly to the curve (one known as local to make erudite) surface.

As the pointer carriage and the compass come to us from China, it can be said that "the sense of orientation comes from the East".

I let to you imagine the rest.

The Team

Luc Macot, physicist
Valmont Veilleux, carpenter
François Raymond, philosopher and photographer

Sources

Gilford Nelson Baronet: Gears, Quebec Sciences, Vol 10 No 2 (November 71) pp 14-15

A site which provided a clear schema of the Mongolian mechanism (probably more spread); in this assembly with twelve gears, the differential functions on a vertical axis:

www.drgears.com/gearterms/terms/southpointingchariots.htm

For the mechanics one of the differential:

www.motorlegend.com/new/technique/differentiel/
barjot-4x4.ovh.org/autres_pages/Technique/Tech_veh_dif.shtml

For the problems of navigation:

Ahmed Djebbar: Une histoire de la science arabe, Point Sciences inédit No 144, Seuil, Paris 2001

Nouveau cours de navigation des Glénans, Seuil, Paris 1972, p 635-638.

For the geometric properties of the pointer carriage:

M Santander: The chinese South-Seeking chariot: A simple mechanical device for visualizing curvature and parallel transport, Am. J. Phys., Vol 60, September 1992, pp 782-787.

This text is a crude translation of the French original found at

www.colvir.net/prof/luc.macot/articles/chariot/

Thanks to Thomas Trouchard for proofreading

The Essence of Orientation - the Pictures

The atom of the mechanism: Two gears

The directions of rotation of two adjacent gears are imposed by the movement of the teeth of the one gear, which drive the teeth of the other gear. Imagine that one of the two gears turns and imagine how it drives the other one.

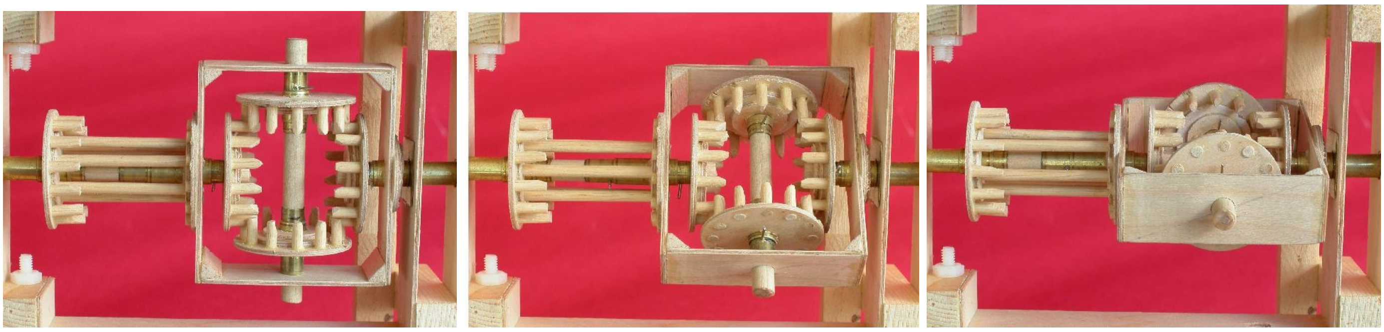
The reverser

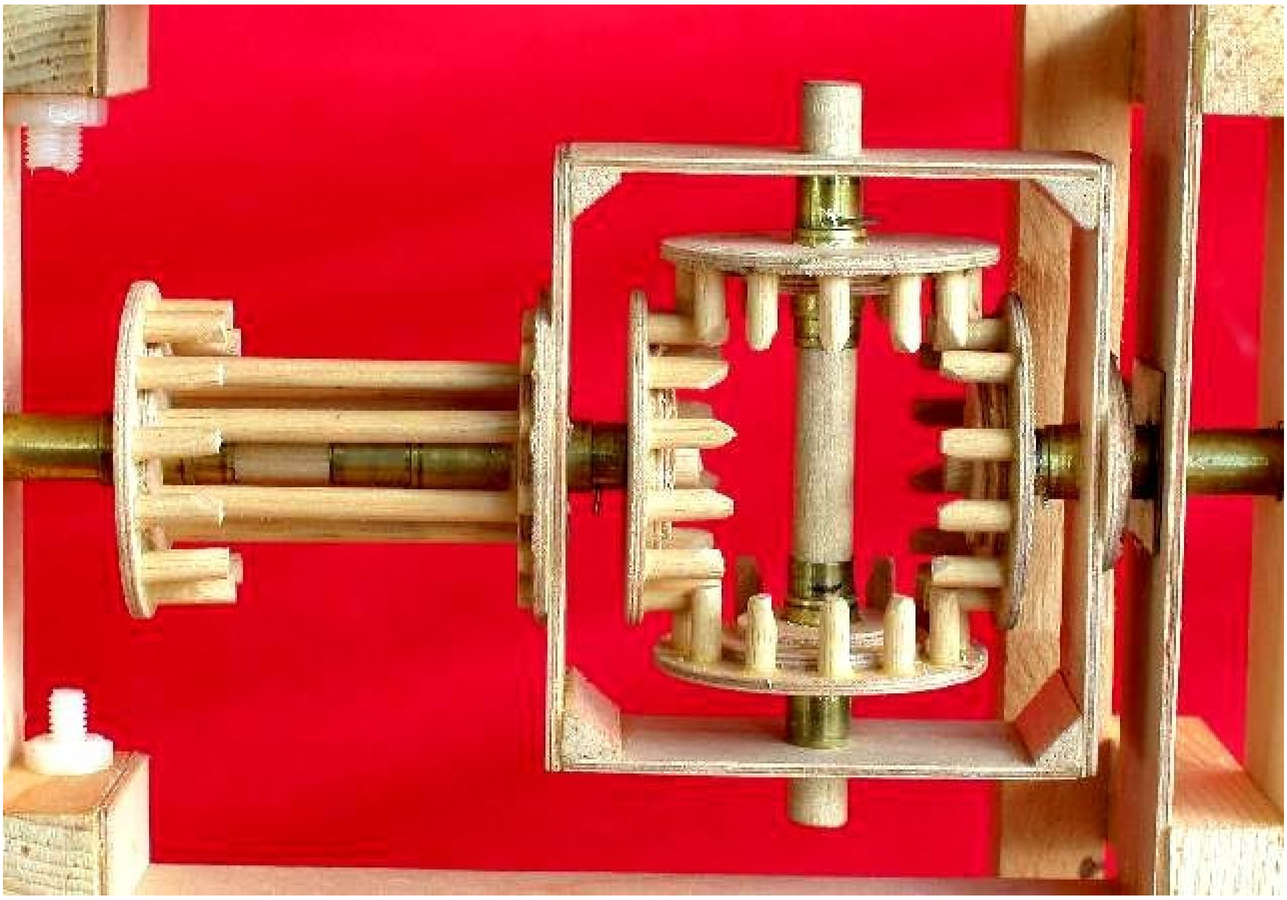
Assume that in the following assembly the vertical axis (carrying the horizontal gear) in the centre of the figure is fixed to the chassis:

If the left gear turns in a direction, the gear on the right-hand side will turn in the opposite direction.

The differential

Here the vertical axis is fixed to a framework (called cage) which in turn can swivel in the chassis:



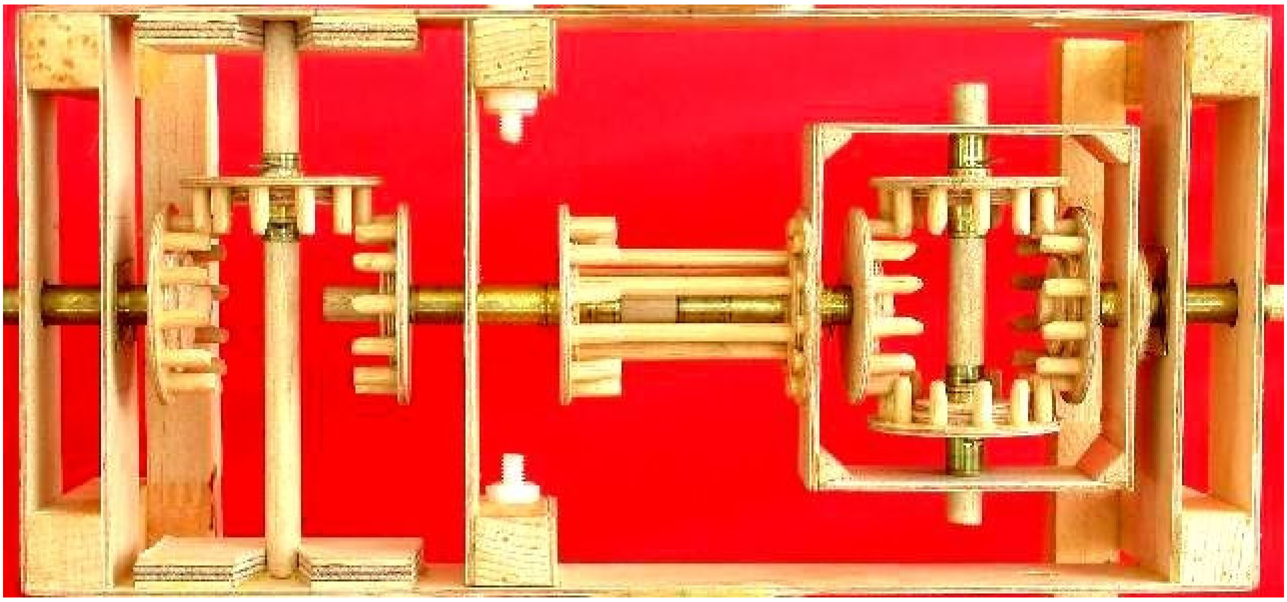


The two gears on the vertical axis must be symmetrical. Concentrate on the four gears on the right of the figure. It has two vertical gears mounted on horizontal axles, these gears are called the "planetary gears"; there are two horizontal gears mounted on a vertical axle but they turn freely on this axle; these gears are called the "satellites".

Visualize the following assertions:

- If both planetary gears turn in the same direction and at the same speed, the satellites will force the cage to turn.
- If both planetary gears turn in opposite directions but at the same speed, the two satellites will also turn in opposite directions at the same speed and the cage does not turn.
- If both planetary gears turn in opposite directions and at different speeds, the cage cannot stay fixed.

The basic mechanism, almost complete:

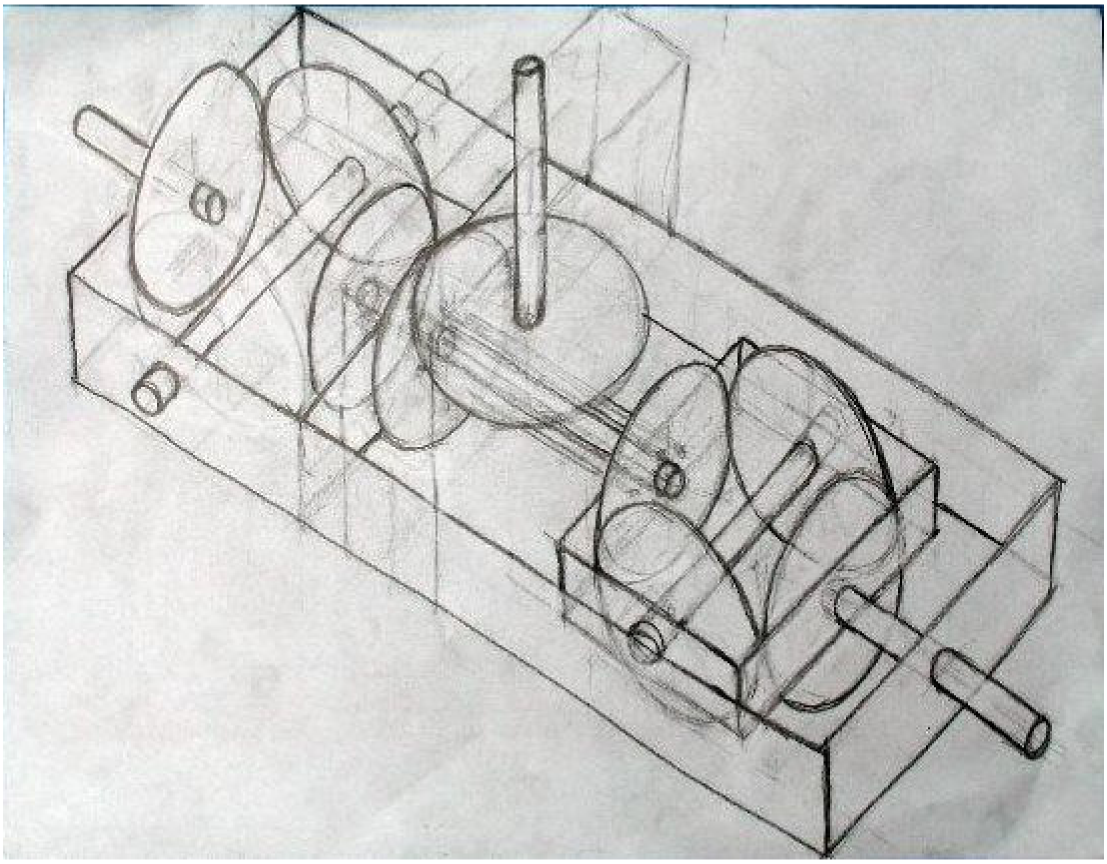


Fix a wheel to the leftmost axle in the figure and do the same to the axle on the right-hand side.

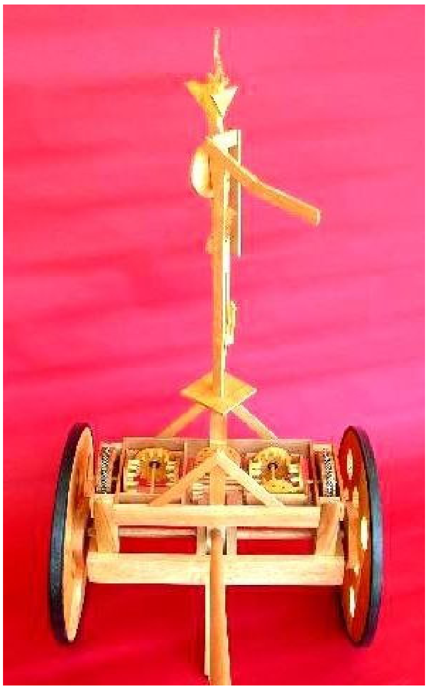
If the carriage advances in a straight line, the two wheels turn at the same speed, so that both planetary wheels of the differential turn (because of the reverser of the rotation of the left wheel) in opposite direction, and the differential remains fixed.

If the wheels of the carriage do not turn at the same speed (for example because the carriage describes a curved trajectory), both planetary gears turn in opposite directions but at different speeds, so that the differential swivels around its axle.

Here the eighth gear comes into the game; it is the gear meshing with the differential, a little left of the centre of the figure. It is used to bring the rotation of the differential back to the pointer, as in this drawing:



If the carriage follows a curved trajectory (in a plane) the pointer will compensate for the change of direction of the carriage. It needs to assured that the compensation of the pointer is equal to the change of direction of the carriage. One simple solution, that one can eventually find by experiment, is that the distance between the wheels is exactly equal to the diameter of the wheels; this is the case with the model (with the small dressed doll), this is also the case with the majority of the known machines. If the distance between the wheels is not equal to their diameter, to make a mechanism operational, a little calculation is to be done; this is the case with the machine appearing on a red background.



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